Logic Grids In The Classroom

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MATH 1120A/B - Fundamental Concepts in Mathematics

Idea

Fundamental to mathematics is the notion of proof, founded on logic and exact reasoning. The two Achilles' heels of first year math students are inexperience and lack of logical thinking. While the first is not easily dealt with, the second can be remedied by introducing students to the notion of mathematical proof early and often.

For first year students, proofs are complicated things that justify the theorems they use and take for granted. Little, if any, thought goes into reading proofs (unless they're going to be on the final) and they are thought of as "those things in the book I don't understand but the teacher recommended I read." They often have little idea of the research, experimentation and steps one takes to form a genuine math proof. This can lead to difficulties for them in later courses and careers.

The activity presented below is based around a simple puzzle and uses logic to turn a set of clues in plain english into a complete result. It asks students to document the steps they take to solve the puzzle, thus creating a proof using their own words but based solely on logic and reasoning.

The Activity

The activity is based around solving a logic puzzle by using a logic grid. In each puzzle students will be given a set of different categories, and an equal number of elements within each category. The aim is to deduce which options are connected by using clues given in plain english. Each puzzle has a unique solution, and can be solved without resorting to guessing.

As the students work through the clues, they are encouraged to use the grids provided, placing a × if two elements cannot be related, and a ✓ if they are. The grids are set up so that each pair of categories is cross-referenced, allowing the students to compare and correlate all the clues. At each step of their deduction, they will record their reasoning, in their own words. Taken together, this reasoning forms a proof of their own making!

For example, we might have a simple puzzle with categories, Name and Age, and three elements in each category, Alice, Bob, Carl and 20, 30, 50.

One of the clues might be something along the lines of, "Bob is older than Alice". From there, one might deduce that Alice cannot be the oldest and likewise Bob can't be the youngest. So they might place an × in the (Alice, 50) and (Bob, 20) boxes, and write down their reasoning:

Names

Age	Alice	Bob	Carl
20		×	
30			
50	×		

Since Bob is older than Alice, he can't be the youngest, as Alice is younger, so I've put an × in the (Bob, 20) position.

Since Bob is older than Alice, she can't be the oldest, as Bob is older, so I've put an × in the (Alice, 50) position.

Their deductions from the clues will build up and eventually they will be able to fill in the entire grid, sorting out which elements belong together. This allows them to create their very own math proof, all by themselves!

The assignment is individual, but students can easily compare with each other thanks to the visual nature of the grids. This allows it to be a somewhat cooperative assignment, comparing results to keep one another on track, but still having to write up their own deductions in their own words. The written nature of the proofs makes this a great assignment to discuss with fellow students, but hard to cheat on.

Key Concepts and Outcomes

The goal of the activity is to gently introduce students to the use of logic and reasoning in coming to new conclusions. In math, such a process, when written out, is called a proof, but in this activity we have disguised it as a simple word puzzle. More specifically, this activity is geared toward teaching the explorative nature of creating a proof, to emphasize that they don't come into existence fully formed, like one would find in a textbook. This lends an air of authenticity to the assignment.

The learning outcomes of this activity are:

- Students will be able to apply logic skills taught in class to determine the correct solution to the puzzle, using the grid as a guide.
- Students will create a proof of the resulting solution, starting with the clues and using step-by-step reasoning to arrive at the result.

Use

The activity itself has great flexibility and scalability. This allows it to be adapted to any setting where students are learning the value of proof and reasoning, not just MATH 1120.

- For more of a challenge (in an upper year logic course) the activity can be scaled up with more categories, more elements per category and more nuanced clues.
- If students are showing a real aptitude for solving the logic grids, flip the activity around and have them design their own. Pair up the students and have them solve and critique each other's puzzle.
- If your course calls for more formal mathematics, tell the students they must first translate the clues into first order logic sentences (defining appropriate relations as well) and then carry out their proofs using first order deduction.

Due to the flexibility and variability logic grids have, the exercise can take any desired length of time. Ranging from five-ten minutes to complete for a small puzzle (say, 3 categories and 4 elements in each) to a couple of hours with a larger grid puzzle (for example, 7 categories and 10 elements in each category).

To employ this activity in class or as a take-home assignment, one needs only to print out copies of a logic grid puzzle. These puzzles can be found online, either from a website with a bank of such puzzles, or online sites that allow you to generate your own. Intrepid instructors may wish to sit down and create their own with whatever categories, elements and clues they wish. There are also online resources to teach one how to do this. One example is the logic puzzle website, http://www.logic-puzzles.org/

All in all, this activity is focused at tackling a key concept in, not just math, but higher order thinking in a playful and engaging manner.

Sample Logic Grid Puzzle

For your convenience, I have included a very simple example of a logic grid puzzle to enhance the explanation in the activity proposal.

Four students meet at a teaching conference in Ottawa. To their surprise they find that each of them is from a different Canadian province and has a different age. The students are, Sarah, Greg, Michael and Heather. Each of them is from one of, Ontario, Manitoba, Alberta or Quebec, and their ages range from 19 to 22 years old.

From the clues below, use the logic grid to identify the province of origin and the age of each of the students at the conference. For each step in your deduction, write an accompanying justification.

The implication of the first clue has been filled in for you as an example.

		Names				Provinces			
		Sarah	Greg	Michael	Heather	Ontario	Manitoba	Alberta	Quebec
Age	19			×					
	20			×					
	21			×					
	22	×	×	✓	×				
Provinces	Ontario								
	Manitoba								
	Alberta								
	Quebec								

Clues

1. Michael is 22 years old.
2. The four people are Greg, the Manitoba resident, the 22 year old and the student from Quebec.
3. The Ontario native is 1 year older than the person from Manitoba.
4. Sarah is older than Heather.
5. The student who is 19 years old is either the from Ontario or Manitoba.