Mathematical Cookbook

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For any math courses at all, but examples are from linear algebra, calculus and statistics.

Idea

Carl Friedrich Gauss once said that "Mathematics is the queen of the sciences." Mathematics is a core part of every science and hence every science class. Logic, specifically step by step deduction, is the foundation of all mathematical thought and activity. So it is clear that understanding the conditions and applications of math results is vital to successful understanding in science classes.

This activity not only highlights the basic parts of logic (starting with preconditions, using deduction and argument, and reaching a conclusion) but it also creates an effective review of notes for a test or exam. At the end of our activity students will have a whole batch of logic recipes that they can easily refer to when studying or completing assignments, both in class and in the future.

The Activity

Students in this activity are asked to take a look through their notes or their textbook and to identify a proof, explanation, or algorithm they think is noteworthy. Their rationale can vary. Maybe they like the result, maybe they think the result is going to be on the final, or maybe they think it will come up a lot in their homework questions.

Once a suitable choice is made, the student is then asked to read the result and disect it into the following parts: the assumptions or preconditions, the conclusions or outcomes, and the logical deductions or explanation. Having divided the result into those categories, they record the result on an index card and are free to stylize the result in the manner of a cookbook, with the ingredients being the preconditions, the result being the 'dish' being cooked up, and the logical arguments are the step-by-step cooking instructions.

Students are encouraged to add their own flair to the recipes they record. I've seen students highlight important parts of the results, add in historical facts as "Chef's notes", and even bring a small box to hold all their index cards, like one would find in a home kitchen.

Key Concepts and Outcomes

This activity is designed to help students understand the components of logical thought and how premises and logical inference combine to form all the mathematical results we know.

The learning outcomes for this activity are:

- Students will be able to analyse important results and algorithms and identify their premises, applications, and conclusions.
- Students will be able to explain and apply the theorems and results which they choose for their cookbook.

Use

This activity is designed to take only a few minutes, which makes it ideal to do at the end of a class as part of a review of that day's theorems and algorithms. This is similar to other classroom activities like muddiest point and minute essay in that they are quick activities that have the student immediately confront the day's lesson. Doing it frequently will train students to create similar cookbooks for other classes and future math courses.

This sort of exercise can be applied to many situations, but most prominantly, it is a good way of having students prepare for exams (midterm, final). You can even encourage them to 'swap recipes' as a way to see what emphasis other students are putting on different results and algorithms.

Sample Receipes

A sample recipe created by my wife for her second year 'Statistics for Psychology' course:

Central Limit Theorem

Ingredients:
- A probability distribution (not necessarily normal)
- Mean, μ, and standard deviation, σ, for the distribution
- A sample size, n, greater than 30

Instructions: If we take the 'distribution of sample means' for the distribution that we are given, the Central Limit Theorem tells us the mean and standard deviation for the 'sample means'. The mean will be $M = \mu$ and the standard deviation will be $s = \sigma/\sqrt{n}$.

Chef's Notes: The sample means are different from the mean of our distribution.

A recipe for an introductory linear algebra proof:

$\vec{u} \times k\vec{u} = \vec{0}$

Ingredients:
- A three dimensional vector, $\vec{u} \in \mathbb{R}^3$
- A scalar value, $k \in \mathbb{R}$

Instructions: First write the coordinates of $\vec{u} = [u_1, u_2, u_3]$. Multiply the coordinates by k, $[ku_1, ku_2, ku_3]$. Perform the cross product formula on the coordinates, $[u_2(ku_3) - (ku_2)u_3, u_3(ku_1) - (ku_3)u_1, u_3(ku_1) - (ku_3)u_1]$. Finally, use the properties of multiplication to show that each coordinate is 0.

A recipe in which key parts have been highlighted, this recipe is how to apply the Intermediate Value Theorem from calculus:

Intermediate Value Theorem

Ingredients:
- A function, f(x)
- A closed interval, [a, b]
- A target value, T

Instructions: First show that f is continuous on the interval [a, b]. Then show either, f(a) < T < f(b) or f(a) > T > f(b). By the Intermediate Value Theorem, we can conclude that there is a number, c, so that a < c < b and f(c) = T.